

Sketch the following rational functions and identify all key information for the given function.

1. $h(x) = \frac{x^2 - 4x - 5}{x^2 - 1} = \frac{(x-5)(x+1)}{(x-1)(x+1)} = \frac{x-5}{x-1}$ $\frac{-6}{-2}$

a. Vertical Asymptote(s): $x=1$

b. Horizontal Asymptote: $y=1$

c. Hole(s): $(-1, 3)$

d. Domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

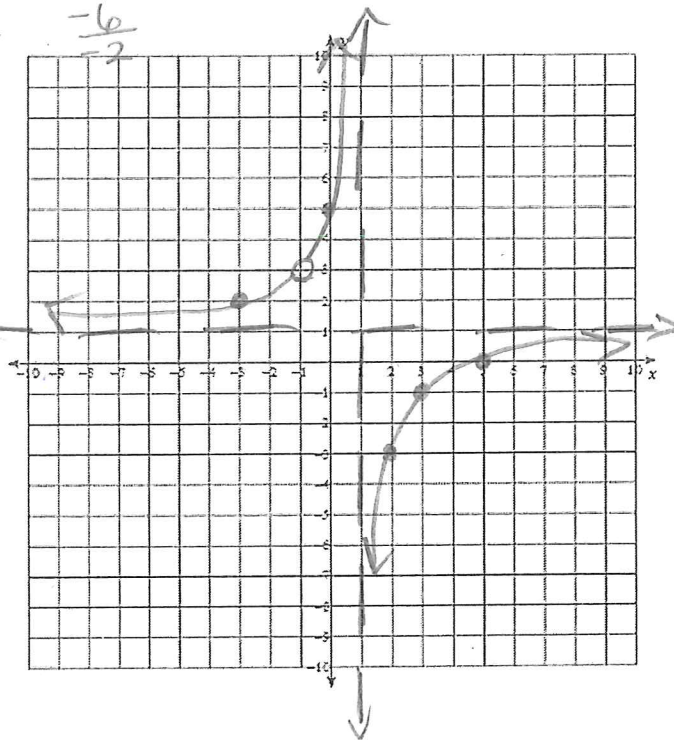
e. Range: $(-\infty, 1) \cup (1, 3) \cup (3, \infty)$

f. x-intercept(s): $(5, 0)$

g. y-intercept: $(0, 5)$

h. End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ 1

As $x \rightarrow \infty, f(x) \rightarrow$ 1



2. $h(x) = \frac{x}{x^2 - 2x - 3} = \frac{x}{(x-3)(x+1)}$

a. Vertical Asymptote(s): $x=3, x=-1$

b. Horizontal Asymptote: $y=0$

c. Hole(s): none

d. Domain: $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

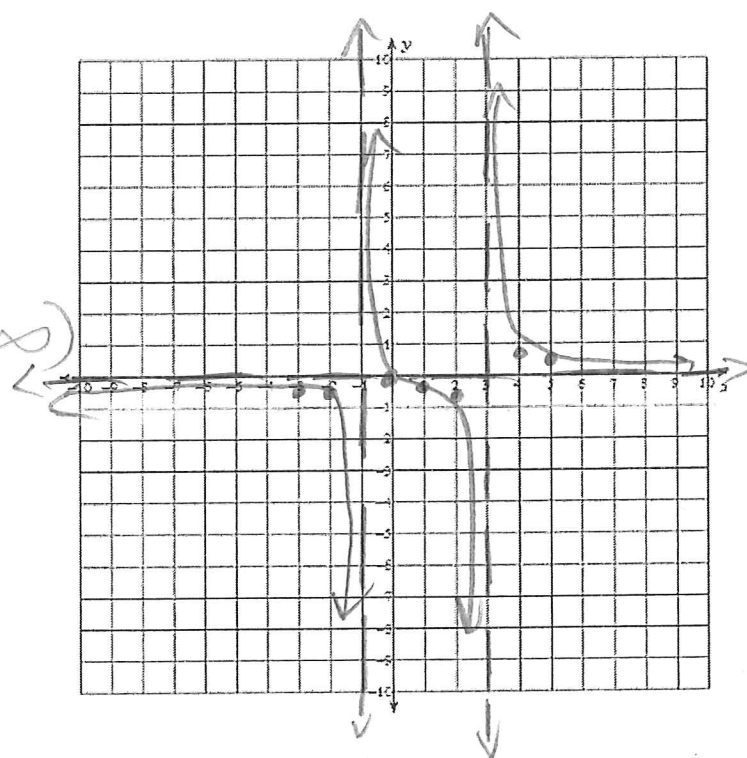
e. Range: $(-\infty, \infty)$

f. x-intercept(s): $(0, 0)$

g. y-intercept: $(0, 0)$

h. End behavior: As $x \rightarrow -\infty, f(x) \rightarrow$ 0

As $x \rightarrow \infty, f(x) \rightarrow$ 0



Solve the following rational equations and identify any extraneous solutions.

$$3.) \frac{1}{n-4} + \frac{n}{n-2} = \frac{2}{n^2-6n+8}$$

$$(n-4)(n-2) \left(\frac{1}{n-4} + \frac{n}{n-2} = \frac{2}{(n-4)(n-2)} \right)$$

$$n-2 + n(n-4) = 2$$

$$n-2 + n^2 - 4n = 2$$

$$n^2 - 3n - 4 = 0$$

$$(n-4)(n+1) = 0$$

$$n = \boxed{-1}$$

$$5.) \frac{m-3}{m-4} + \frac{5}{m^2-3m-4} = 1$$

$$(m-4)(m+1) \left(\frac{m-3}{m-4} + \frac{5}{(m-4)(m+1)} = 1 \right)$$

$$(m-3)(m+1) + 5 = (m-4)(m+1)$$

$$m^2 - 2m - 3 + 5 = m^2 - 3m - 4$$

$$\boxed{m = -6}$$

$$4.) \frac{x+8}{x-6} = \frac{-4}{1}$$

$$x+8 = -4(x-6)$$

$$x+8 = -4x+24$$

$$5x = 16$$

$$\boxed{x = 16/5}$$

$$6.) \left(\frac{2}{x} - \frac{4}{x+1} = 3 \right) x(x+1)$$

$$2(x+1) - 4(x) = 3x(x+1)$$

$$2x+2-4x = 3x^2+3x$$

$$0 = 3x^2 + 5x - 2$$

$$0 = (3x-1)(x+2)$$

$$\boxed{x = 1/3, -2}$$

7.) Circle all that apply to the function $h(x) = \frac{x^3 - 5x^2 + 6x}{-4x^2 - 4x + 24} = \frac{x(x^2 - 5x + 6)}{-4(x^2 + x - 6)}$

a. $(0, 0)$ is an y-intercept

b. $(0, 0)$ is an x-intercept

c. $(2, 0)$ is an x-intercept

d. $(3, 0)$ is an x-intercept

e. $(-3, 0)$ is an x-intercept

f. $(2, 1/10)$ is a hole

g. $x = -3$ is a vertical asymptote

h. $x = 3$ is a vertical asymptote

$\frac{x(x-3)(x-2)}{-4(x+3)(x-6)}$
 HOLE: $(2, \frac{1}{10})$

Refer to the rational function $g(x) = \frac{x^2 + 7x + 12}{x^2 - 9}$ to answer problems 8 through 11.

8.) The horizontal asymptote of $g(x)$ is _____

9.) A vertical asymptote(s) of $g(x)$ is _____

a. $x = 1$

b. $y = 1$

c. $x = 3$

d. $y = 3$

$\frac{(x+4)(x+3)}{(x-3)(x+3)}$
 $(-3, -\frac{1}{6})$

a. $x = 1$

b. $y = 1$

c. $x = 3$

d. $y = 3$

10.) A hole of $g(x)$ is located at _____

11.) The domain of $g(x)$ is _____

a. $(-3, 0)$

b. $(-3, -\frac{1}{6})$

c. $(3, 0)$

d. $(3, -\frac{1}{6})$

a. $(-\infty, 3) \cup (3, \infty)$

b. $(-\infty, -3) \cup (-3, \infty)$

c. $(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$

d. $(-\infty, \infty)$

