

Identify the key information for the following rational functions, then graph.

1.)  $f(x) = \frac{x^2 + 3x}{x^2 + x - 6} \cdot \frac{x(x+3)}{(x+3)(x-2)} \cdot \frac{x}{x-2}$

Vertical Asymptote:  $x = 2$

Horizontal Asymptote:  $y = 1$

Hole(s):  $(-3, 3/5)$   $\frac{-3}{-3-2} = \frac{-3}{-5} = 3/5$

Domain:  $(-\infty, -3) (-3, 2) (2, \infty)$

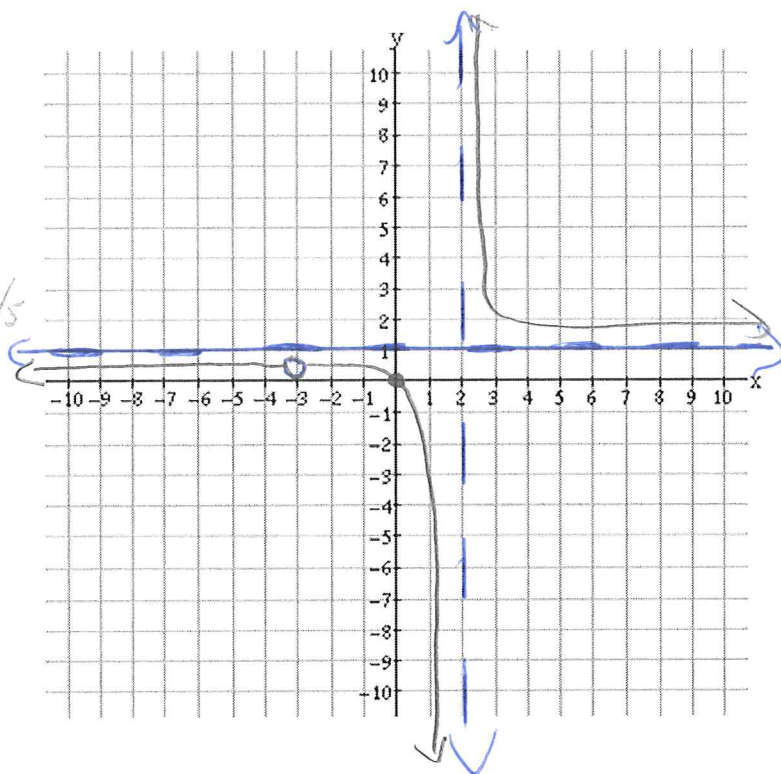
Range:  $(-\infty, 3/5) (3/5, 1) (1, \infty)$

x-intercept(s):  $(0, 0)$   $0 = x$

y-intercept:  $(0, 0)$

End Behavior: As  $x \rightarrow -\infty, f(x) \rightarrow$   $1$

As  $x \rightarrow \infty, f(x) \rightarrow$   $1$



2.)  $f(x) = \frac{x^2 - 5x + 4}{x^2 - 16} \cdot \frac{(x-4)(x-1)}{(x-4)(x+4)} \cdot \frac{x-1}{x+4}$

Vertical Asymptote:  $x = -4$

Horizontal Asymptote:  $y = 1$

Hole(s):  $(4, 3/8)$   $\frac{4-1}{4+4} = \frac{3}{8}$

Domain:  $(-\infty, -4) (-4, 4) (4, \infty)$

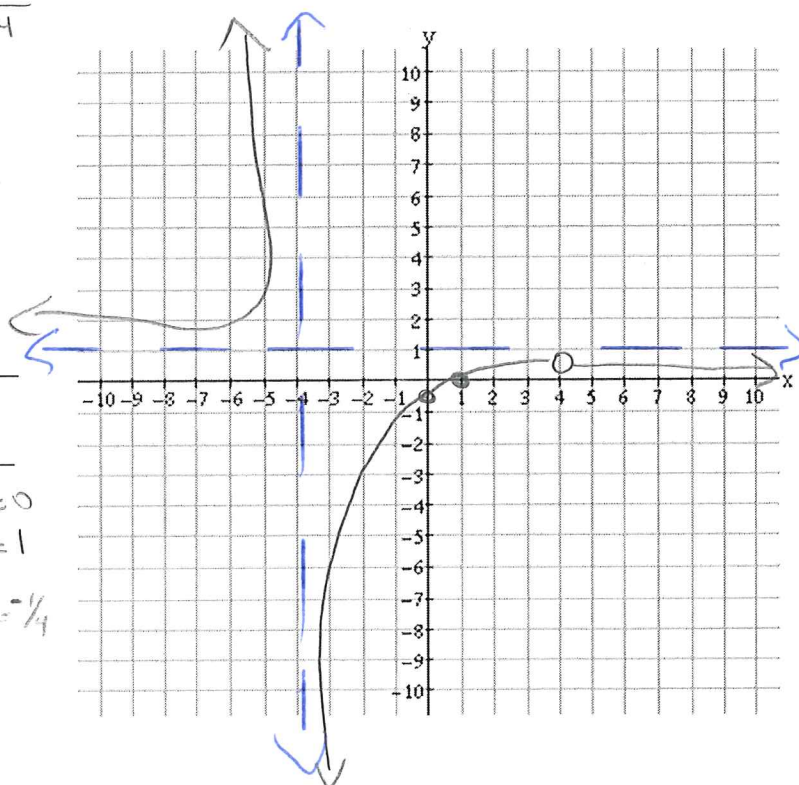
Range:  $(-\infty, 5/8) (5/8, 1) (1, \infty)$

x-intercept(s):  $(1, 0)$   $x-1=0$   
 $x=1$

y-intercept:  $(0, -1/4)$   $\frac{0-1}{0+4} = -1/4$

End Behavior: As  $x \rightarrow -\infty, f(x) \rightarrow$   $1$

As  $x \rightarrow \infty, f(x) \rightarrow$   $1$



$$3.) f(x) = \frac{x^2 - 25}{x + 5} \quad \frac{(x-5)(x+5)}{(x+5)} \quad (x-5)$$

Vertical Asymptote: none

Horizontal Asymptote: none

Hole(s):  $(-5, -10)$   $x+5=0 \rightarrow -5-5=-10$

Domain:  $(-\infty, -5)$   $(-5, \infty)$

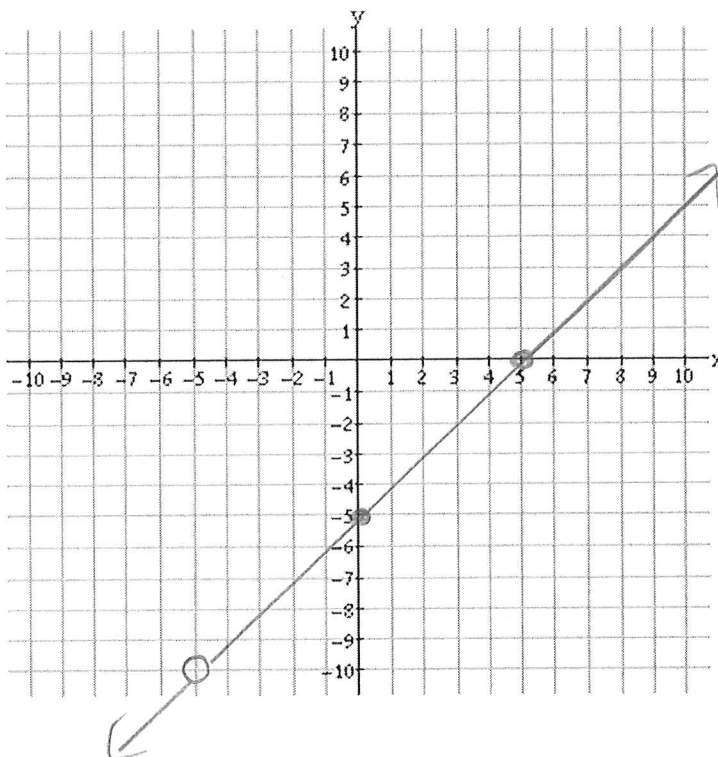
Range:  $(-\infty, -10)$   $(-10, \infty)$

x-intercept(s):  $(5, 0)$   $0=x-5$   
 $5=x$

y-intercept:  $(0, -5)$   $0-5=-5$

End Behavior: As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow -\infty$

As  $x \rightarrow \infty$ ,  $f(x) \rightarrow \infty$



$$4.) f(x) = \frac{x}{2x^2 - 4x} \quad \frac{x}{2x(x-2)} \quad \frac{1}{2(x-2)}$$

Vertical Asymptote:  $x=2$

Horizontal Asymptote:  $y=0$

Hole(s):  $(0, -1/4)$   $\frac{1}{2(0-2)} = -1/4$

Domain:  $(-\infty, 0)$   $(0, 2)$   $(2, \infty)$

Range:  $(-\infty, -1/4)$   $(-1/4, 0)$   $(0, \infty)$

x-intercept(s): none

y-intercept: none

End Behavior: As  $x \rightarrow -\infty$ ,  $f(x) \rightarrow 0$

As  $x \rightarrow \infty$ ,  $f(x) \rightarrow 0$

