

Chapter 2 Solving polynomials
with imaginary solutions

Write the following in factored form then list all the zeros.

1.) $f(x) = x^4 - 4x^3 + 7x^2 - 12x + 12$

$$2 \begin{array}{r|rrrrr} & 1 & -4 & 7 & -12 & 12 \\ & & 2 & -4 & 6 & -12 \\ \hline & 1 & -2 & 3 & -6 & 0 \end{array}$$

$$x^3 - 2x^2 + 3x - 6$$

$$x^2(x-2) \quad 3(x-2)$$

$$(x^2 + 3)(x-2)$$

$$x^2 = -3 \quad x = 2$$

$$x = \pm i\sqrt{3}$$

Factored Form: $(x-2)(x-2)(x^2+3)$

Zero(s): $x = 2, 2, \pm i\sqrt{3}$

3.) $f(x) = x^3 - 10x^2 + 44x - 69$

$$3 \begin{array}{r|rrrr} & 1 & -10 & 44 & -69 \\ & & 3 & -21 & 69 \\ \hline & 1 & -7 & 23 & 0 \end{array}$$

$$x^2 - 7x + 23$$

$$\frac{7 \pm \sqrt{(-7)^2 - 4(1)(23)}}{2(1)}$$

$$\frac{7 \pm \sqrt{-43}}{2}$$

Factored Form: $(x-3)(x^2-7x+23)$

Zero(s): $3, \frac{7 \pm i\sqrt{43}}{2}$

2.) $f(x) = 4x^4 + 26x^3 - 8x^2 + 39x - 21; -7$

$$-7 \begin{array}{r|rrrrr} & 4 & 26 & -8 & 39 & -21 \\ & & -28 & 14 & -42 & 21 \\ \hline & 4 & -2 & 4 & -3 & 0 \end{array}$$

$$4x^3 - 2x^2 + 4x - 3$$

$$2x^2(2x-1) \quad 3(2x-1)$$

$$(2x-1)(2x^2+3)$$

$$x = \frac{1}{2}$$

$$x^2 = -\frac{3}{2}$$

$$x = \pm i\sqrt{\frac{3}{2}}$$

Factored Form: $(x+7)(2x^2+3)(2x-1)$

Zero(s): $-7, \frac{1}{2}, \pm i\sqrt{\frac{3}{2}}$

4.) $f(x) = x^4 - 4x^3 - 2x^2 - 12x - 15$

$$5 \begin{array}{r|rrrrr} & 1 & -4 & -2 & -12 & -15 \\ & & 5 & 5 & 15 & 15 \\ \hline & 1 & 1 & 3 & 3 & 0 \end{array}$$

$$x^3 + x^2 + 3x + 3$$

$$x^2(x+1) \quad 3(x+1)$$

$$(x^2+3)(x+1)$$

$$x^2 = -3 \quad x = -1$$

$$x = \pm i\sqrt{3}$$

Factored Form: $(x-5)(x+1)(x^2+3)$

Zero(s): $5, -1, \pm i\sqrt{3}$

$$5.) f(x) = x^5 + 3x^4 - 4x - 12 \quad -3$$

$$-3 \left| \begin{array}{cccc} 1 & 3 & -4 & -12 \\ & -3 & 0 & 12 \\ \hline & 1 & 0 & -4 & 0 \end{array} \right.$$

$$x^4 - 4$$

$$(x^2 + 2)(x^2 - 2)$$

$$x^2 = -2 \quad x^2 = 2$$

$$x = \pm i\sqrt{2} \quad x = \pm \sqrt{2}$$

Factored Form: $(x+3)(x^2-2)(x^2+2)$

Zero(s): $-3, \pm\sqrt{2}, \pm i\sqrt{2}$

$$7.) f(x) = 2x^4 - 11x^3 + 11x^2 - 176x - 336 \quad 7$$

$$7 \left| \begin{array}{cccccc} 2 & -11 & 11 & -176 & -336 \\ & 14 & 21 & 224 & 336 \\ \hline & 2 & 3 & 32 & 48 & 0 \end{array} \right.$$

$$2x^3 + 3x^2 + 32x + 48$$

$$x^2(2x+3) + 16(2x+3)$$

$$(x^2 + 16)(2x + 3)$$

$$x^2 = -16$$

$$x^2 = \pm\sqrt{-16}$$

$$\pm 4i$$

Factored Form: $(x-7)(2x+3)(x^2+16)$

Zero(s): $7, -3/2, \pm 4i$

$$6.) f(x) = 2x^3 - x^2 + 2x - 1; \frac{1}{2}$$

$$\frac{1}{2} \left| \begin{array}{cccc} 2 & -1 & 2 & -1 \\ & 1 & 0 & 1 \\ \hline & 2 & 0 & 2 & 0 \end{array} \right.$$

$$2x^2 - 2$$

$$2(x^2 - 1)$$

$$(x-1)(x+1)$$

$$x = \frac{1}{2}$$

$$2x = 1$$

$$(2x - 1) \neq 0$$

Factored Form: $(2x-1)(x-1)(x+1)$

Zero(s): $1/2, 1, -1$

$$8.) f(x) = x^5 - 4x^4 - 4x^3 + 16x^2 - 45x + 180$$

$$-3 \left| \begin{array}{cccccc} 1 & -4 & -4 & 16 & -45 & 180 \\ & -3 & 21 & -51 & 105 & -180 \\ \hline & 1 & -7 & 17 & -35 & 60 & 0 \end{array} \right.$$

$$3 \left| \begin{array}{cccc} 3 & -12 & 15 & -60 \\ & 1 & -4 & 5 & -20 \\ \hline & 1 & -4 & 5 & -20 & 0 \end{array} \right.$$

$$x^3 - 4x^2 + 5x - 20$$

$$x^2(x-4) + 5(x-4)$$

$$(x^2 + 5)(x - 4)$$

Factored Form: $(x+3)(x-3)(x^2+5)(x-4)$

Zero(s): $-3, 3, 4, \pm i\sqrt{5}$