

Fill in the blank with the correct identity.

$$1.) \sin^2 x = \underline{1} - \underline{\cos^2 x}$$

$$8.) \frac{1}{\csc x} = \underline{\sin x}$$

$$2.) \tan^2 x = \underline{\sec^2 x} - \underline{1}$$

$$9.) \frac{\cos x}{\sin x} = \underline{\cot x}$$

$$3.) \csc^2 x = \underline{1} + \underline{\cot^2 x}$$

$$10.) \frac{1}{\cos x} = \underline{\sec x}$$

$$4.) \cos^2 x = \underline{1} - \underline{\sin^2 x}$$

$$11.) \frac{\sin x}{\cos x} = \underline{\tan x}$$

$$5.) \cot^2 x = \underline{\csc^2 x} - \underline{1}$$

$$12.) \frac{1}{\cot x} = \underline{\tan x}$$

$$6.) \sec^2 x = \underline{1} + \underline{\tan^2 x}$$

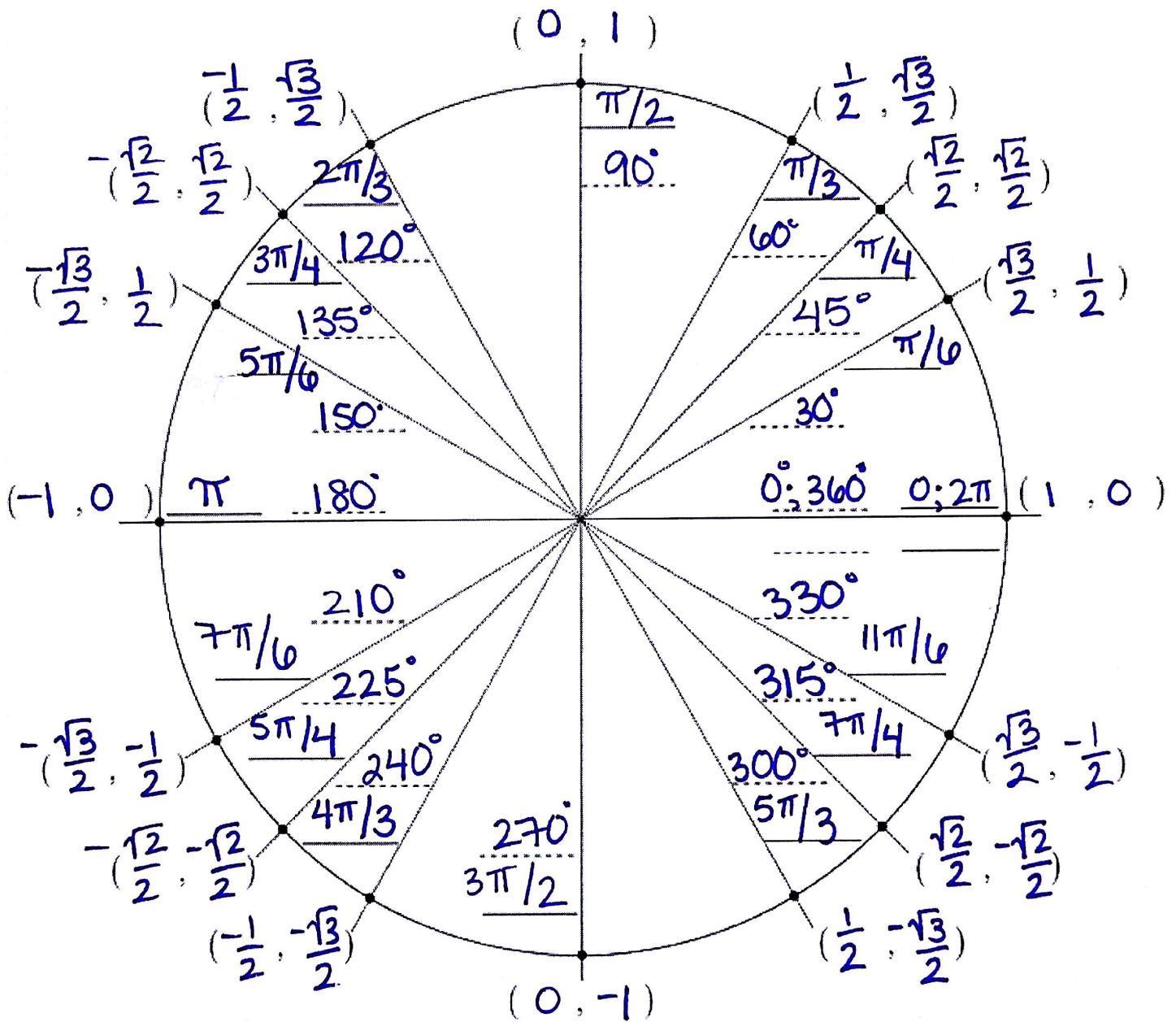
$$13.) \frac{1}{\tan x} = \underline{\cot x}$$

$$7.) \underline{\sin^2 x} + \underline{\cos^2 x} = 1$$

$$14.) \frac{1}{\sec x} = \underline{\cos x}$$

$$15.) \frac{1}{\sin x} = \underline{\csc x}$$

# Unit Circle



Solve each equation over the interval  $[0^\circ, 360^\circ)$ .

16.  $1 - 12\cos^2 x = -5$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \sqrt{\frac{1}{2}} = \pm \frac{1}{\sqrt{2}} = \pm \frac{\sqrt{2}}{2}$$

$$x = 45^\circ, 135^\circ, 225^\circ, 315^\circ$$

17.  $(\sec x + \sqrt{2})(\tan x + \sqrt{3}) = 0$

$$\sec x = -\sqrt{2}$$

$$\tan x = -\sqrt{3}$$

$$\cos x = -\frac{\sqrt{2}}{2}$$

$$x = 135^\circ, 225^\circ, 120^\circ, 300^\circ$$

Solve each equation over the interval  $[0, 2\pi)$ .

18.  $2\sin x \cos x + \sqrt{3} \sin x = 0$

$$\sin x (2\cos x + \sqrt{3}) = 0$$

$$\sin x = 0 \quad \cos x = -\frac{\sqrt{3}}{2}$$

$$x = 0, \pi, \frac{5\pi}{6}, \frac{7\pi}{6}$$

19.  $2\cos^2 x + 3\cos x - 2 = 0$

$$(2\cos x - 1)(\cos x + 2) = 0$$

$$\cos x = \frac{1}{2} \quad \cos x = -2$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

Verify.

20.  $\frac{\sin^2 x}{\cos x} = \sec x - \cos x$

$$\frac{1 - \cos^2 x}{\cos x} = \sec x - \cos x$$

$$\frac{1}{\cos x} - \frac{\cos^2 x}{\cos x} = \sec x - \cos x$$

$$\sec x - \cos x = \sec x - \cos x$$

✓

21.  $\frac{\sin x - \sin^3 x}{\cos x} = \tan x \cos^2 x$

$$\frac{\sin x (1 - \sin^2 x)}{\cos x} = \tan x \cos^2 x$$

$$\frac{\sin x (\cos^2 x)}{\cos x} = \tan x \cos^2 x$$

$$\frac{\sin x}{\cos x} \cos^2 x = \tan x \cos^2 x$$

$$\tan x \cos^2 x = \tan x \cos^2 x$$

✓

State the amplitude, period, and the horizontal and vertical shift for each function. Then graph.

22.  $y = -2 \cos(x - \pi) - 3$

Amplitude = 2

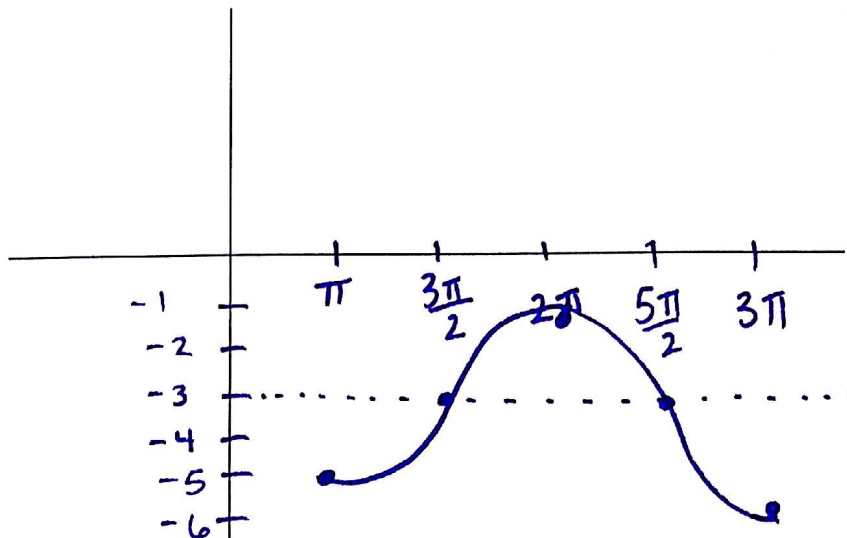
Period =  $2\pi$

H. S. =  $\pi$

V. S. = 3

D:  $(-\infty, \infty)$

R:  $[-5, -1]$



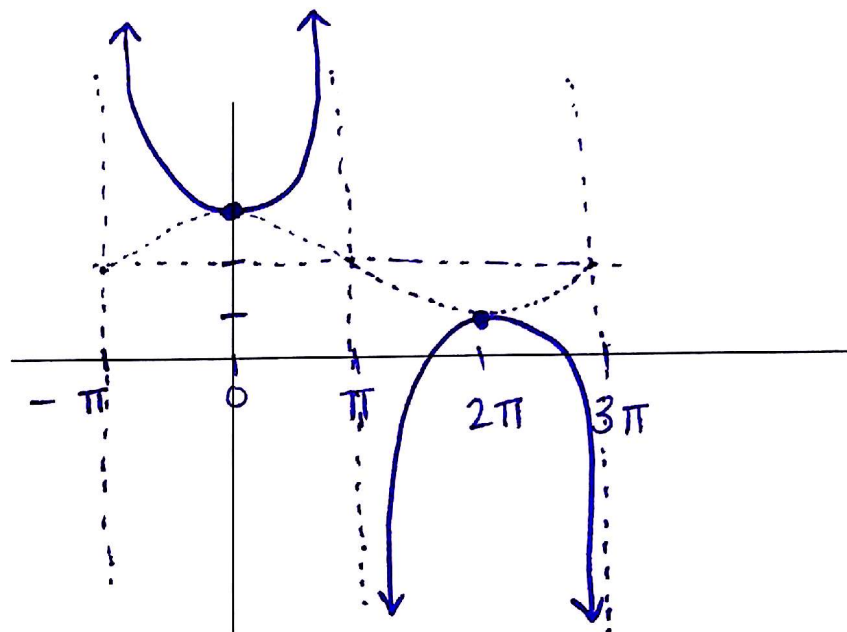
23.  $y = \csc \frac{1}{2}(x + \pi) + 2$

Amplitude = DNE

Period =  $4\pi$

H. S. =  $\pi$

V. S. = 2



24.  $y = 4 \sin 2x - 1$

Amplitude = 4

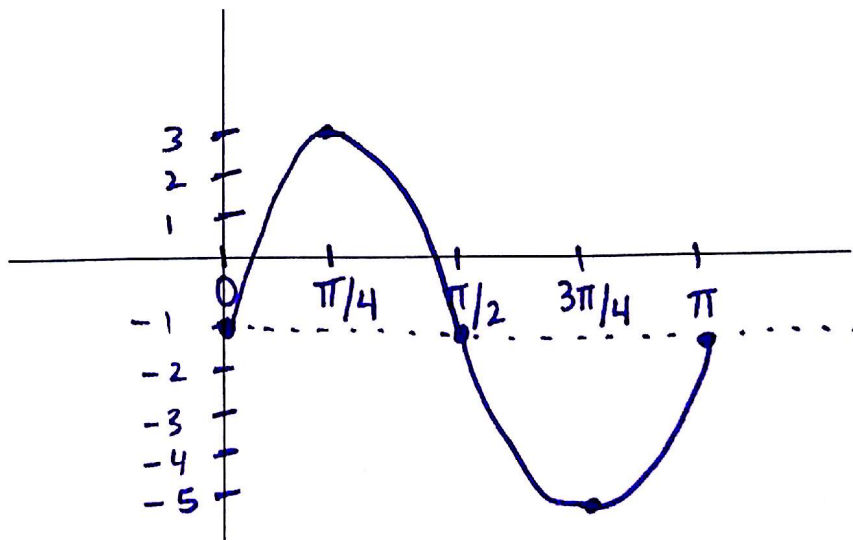
Period =  $\pi$

H. S. = 0

V. S. = 1

D:  $(-\infty, \infty)$

R:  $[-5, 3]$





Solve the triangle. If necessary, round to the nearest hundredths.

25.  $A=103.5^\circ$   $a=41.5$   $b=27.2$

$$\frac{41.5}{\sin 103.5^\circ} = \frac{27.2}{\sin B}$$

$$41.5 \sin B = 27.2 \sin 103.5^\circ$$

$$B = \sin^{-1} \left( \frac{27.2 \sin 103.5^\circ}{41.5} \right)$$

$$B = 39.59^\circ \rightarrow 180^\circ - B = B'$$

$$\frac{41.5}{\sin 103.5^\circ} = \frac{c}{\sin 36.91^\circ}$$

$A=103.5^\circ$ $a=41.5$	<del><math>A'=103.5^\circ</math> <math>a'=41.5</math></del>
$B=39.59^\circ$ $b=27.2$	<del><math>B'=140.41^\circ</math> <math>b'=27.2</math></del>
$C=36.91^\circ$ $c=25.63$	<del><math>C'=-63.91^\circ</math> <math>c'=-</math></del>

27.  $a=80.1$   $b=92.3$   $c=123.8$

$$80.1^2 = 92.3^2 + 123.8^2 - 2(92.3)(123.8)\cos A$$

$$-92.3^2 \quad -92.3^2 \quad -123.8^2$$

$$-123.8$$

$$\frac{-17429.72}{-22853.48} = \frac{-22853.48 \cos A}{-22853.48}$$

$$\cos^{-1}(\uparrow) = 40.3^\circ$$

$$\frac{80.1}{\sin 40.3^\circ} = \frac{92.3}{\sin B}$$

$A=40.3^\circ$	$a=80.1$
$B=48.19^\circ$	$b=92.3$
$C=91.51^\circ$	$c=123.8$

26.  $A=145^\circ$   $b=4$   $c=14$

$$a^2 = 4^2 + 14^2 - 2(4)(14)\cos 145^\circ$$

$$a^2 = 303.745029$$

$$a = 17.43$$

$$\frac{17.43}{\sin 145^\circ} = \frac{4}{\sin B} \rightarrow B = 7.56^\circ$$

$A=145^\circ$	$a=17.43$
$B=7.56^\circ$	$b=4$
$C=27.44^\circ$	$c=14$

28.  $A=55.3^\circ$   $a=22.8$   $b=24.9$

$$\frac{22.8}{\sin 55.3^\circ} = \frac{24.9}{\sin B}$$

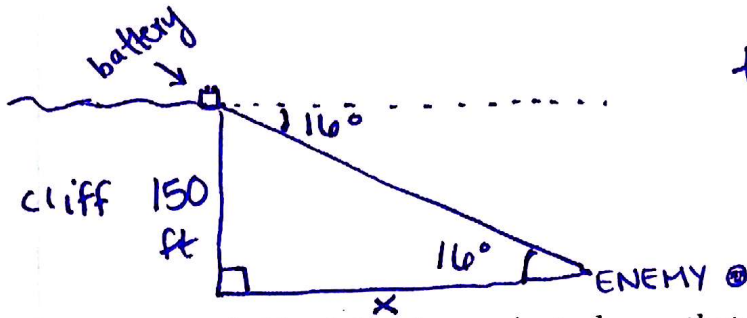
$$B = 63.88^\circ \rightarrow B' = 116.12^\circ$$

$$\frac{22.8}{\sin 55.3^\circ} = \frac{c}{\sin 60.82^\circ} \quad \left| \quad \frac{22.8}{\sin 55.3^\circ} = \frac{c}{\sin 8.58^\circ} \right.$$

$A=55.3^\circ$	$a=22.8$	$A=55.3^\circ$	$a=22.8$
$B=63.88^\circ$	$b=24.9$	$B=116.12^\circ$	$b=24.9$
$C=60.82^\circ$	$c=24.21$	$C=8.58^\circ$	$c=4.14$

**Solve. If necessary, round answers to the nearest hundredths.**

29. An artillery battery is at the top of a 150 foot cliff. If the enemy is located at an angle of depression of  $16^\circ$  from the top of the cliff, how far is the enemy from the base of the cliff?

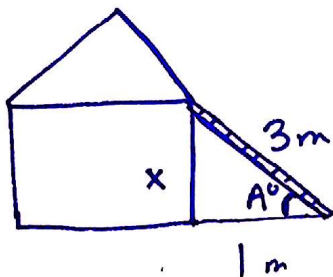


$$\tan 16^\circ = \frac{150}{x}$$

$$x = \frac{150}{\tan 16^\circ}$$

$$x = 523.11 \text{ ft}$$

30. A 3 meter ladder is leaning against a house that is 1 meter from the base of the house.  
 (a) What is the angle of elevation? (b) How far above the ground is the ladder touching the house?



$$\cos A = \frac{1}{3}$$

$$A = 70.53^\circ$$

$$1^2 + x^2 = 3^2$$

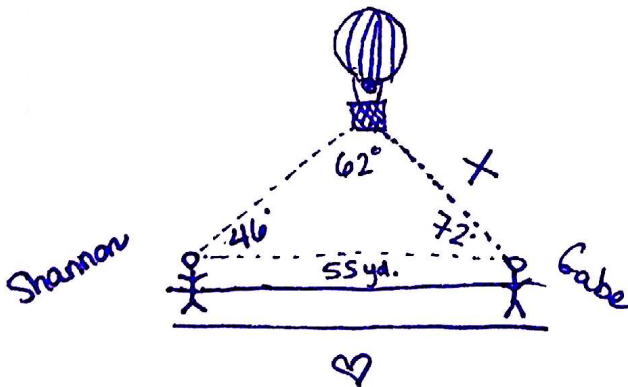
$$1 + x^2 = 9$$

$$x^2 = 8$$

$$x = 2.83$$

$$x = 2\sqrt{2} \text{ ft}$$

31. Shannon and Gabe are walking toward each other on a path through the park. The path runs east-west. A hot air balloon is directly above the path between them. Shannon, sees the balloon when looking east at an angle of elevation of  $46^\circ$ . Gabe, sees the balloon looking west at an angle of elevation of  $72^\circ$ . If they are 55 yards apart, how far is the balloon from Gabe?



$$\frac{55}{\sin 62^\circ} = \frac{x}{\sin 46^\circ}$$

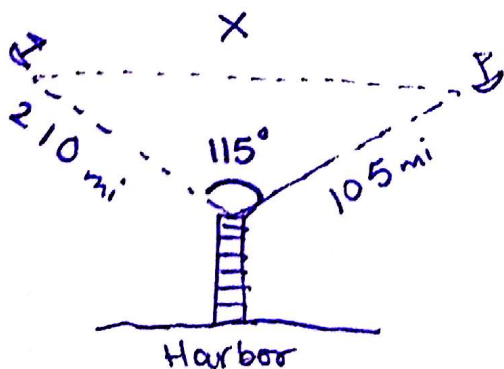
$$x = 44.8 \text{ yds.}$$

32. Two ships leave a harbor together, traveling on courses that have an angle of  $115^\circ$  between them. One ship traveled 105 miles, the other ship traveled 210 miles, how far apart are the ships?

\* remember: we can only use pyth. thm. for  $\perp$   $\Delta$ s

$$x^2 = 105^2 + 210^2 - 2(105)(210)\cos 115$$

$$x = 271.59 \text{ mi}$$





Find the following information for the given angle measure.

33.  $-671^\circ$

Quadrant: I

Reference Angle:  $49^\circ$

+ Co-terminal Angle =  $49^\circ$

- Co-terminal Angle =  $-311^\circ$

34.  $1274^\circ$

Quadrant: III

Reference Angle:  $14^\circ$

+ Co-terminal Angle =  $914^\circ$

- Co-terminal Angle =  $-166^\circ$

Convert to degrees.

35.  $\frac{6\pi}{5} \cdot \frac{180^\circ}{\pi} = \boxed{216^\circ}$

36.  $\frac{2\pi}{9} \cdot \frac{180^\circ}{\pi} = \boxed{40^\circ}$

37.  $-\frac{10\pi}{3} \cdot \frac{180^\circ}{\pi} = \boxed{-600^\circ}$

Convert to radians.

38.  $342^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{19\pi}{10}}$

39.  $-124^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{31\pi}{45}}$

40.  $720^\circ \cdot \frac{\pi}{180^\circ} = \boxed{4\pi}$

Given the following point on the terminal side of angle  $\theta$ , find the six trig functions for  $\theta$ .

41.  $(-9, -40)$   $(-9)^2 + (-40)^2 = r^2$   
 $41 = r$

$\frac{y}{r} \sin \theta = \frac{-40}{41}$   $\csc \theta = \frac{41}{-40}$

$\frac{x}{r} \cos \theta = \frac{-9}{41}$   $\sec \theta = \frac{41}{-9}$

$\frac{y}{x} \tan \theta = \frac{40}{9}$   $\cot \theta = \frac{9}{40}$

42.  $(7, -24)$   $r = 25$

$\sin \theta = \frac{-24}{25}$   $\csc \theta = \frac{25}{-24}$

$\cos \theta = \frac{7}{25}$   $\sec \theta = \frac{25}{7}$

$\tan \theta = \frac{-24}{7}$   $\cot \theta = \frac{7}{-24}$

Find the values of the six trigonometric functions with the given information.

43.  $\sin \theta = \frac{5}{13r}$   $\theta$  in quadrant II  $\frac{5}{13}$   $\frac{12}{-12}$

$\sin \theta = \frac{5}{13}$   $\csc \theta = \frac{13}{5}$

$\cos \theta = \frac{-12}{13}$   $\sec \theta = \frac{13}{-12}$

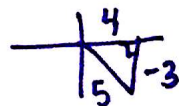
$\tan \theta = \frac{5}{-12}$   $\cot \theta = \frac{-12}{5}$

44.  $\tan \theta = -\frac{3}{4}$   $\frac{3\pi}{2} < \theta < 2\pi$

$\sin \theta = \frac{-3}{5}$   $\csc \theta = \frac{5}{-3}$

$\cos \theta = \frac{4}{5}$   $\sec \theta = \frac{5}{4}$

$\tan \theta = \frac{-3}{4}$   $\cot \theta = \frac{4}{-3}$



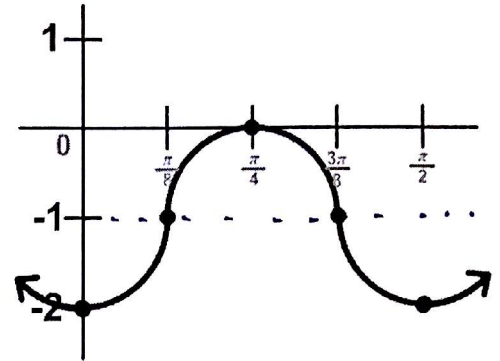
$$\frac{8\pi}{3} = \frac{2\pi}{B} \rightarrow B = 3/4$$

45. Write the equation of the sine function that has an amplitude of 3, period of  $\frac{8\pi}{3}$ , and is shifted up 5.

Equation:  $y = 3 \sin \frac{3}{4}(x) + 5$

46. Write the equation of the graph to the right.

Equation:  $y = -\cos 4(x) - 1$



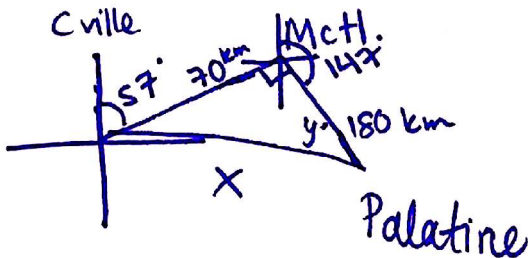
47. Find the period of the function  $y = \tan\left(3x - \frac{\pi}{2}\right) + 5$

$$y = \tan 3\left(x - \frac{\pi}{6}\right) + 5$$

$$\boxed{\pi/3}$$

**Solve.** If necessary, round answers to the nearest hundredths.

48. The bearing from Carpentersville to McHenry is  $57^\circ$  and the bearing from McHenry to Palatine is  $147^\circ$ . If Carpentersville is 70 km from McHenry and McHenry is 180 km from Palatine, how far is Palatine from Carpentersville? What is the measure of the angle formed at Palatine?

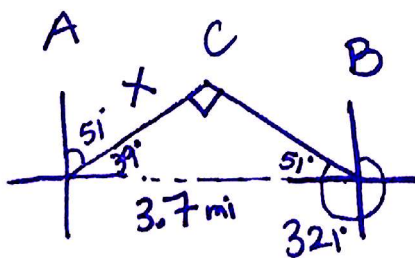


$$70^2 + 180^2 = x^2$$

$$\boxed{193.13 \text{ km} = x}$$

$$\tan y^\circ = \frac{70}{180} \rightarrow \boxed{y = 21.25^\circ}$$

49. Radar stations A and B are on an east-west line, 3.7 miles apart. Station A detects a plane at C, on a bearing of  $51^\circ$ . Station B detects the plane at a bearing of  $321^\circ$ . Find the distance from A to C.



$$\sin 51^\circ = \frac{x}{3.7}$$

$$\boxed{2.88 \text{ mi} = x}$$



Find the exact value for each of the following.

50.)  $\sin \frac{2\pi}{3}$

$$\boxed{\frac{\sqrt{3}}{2}}$$

51.)  $\cos 90^\circ$

$$\boxed{0}$$

52.)  $\tan \frac{5\pi}{4}$

$$-\frac{\frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} = \boxed{1}$$

53.)  $\csc \frac{\pi}{4}$

$$\boxed{\sqrt{2}}$$

54.)  $\cot 330^\circ$

$$\boxed{-\sqrt{3}}$$

55.)  $\sec \frac{3\pi}{2}$

$$\boxed{\text{undefined}}$$

56.)  $\cos \frac{5\pi}{3}$

$$\boxed{\frac{1}{2}}$$

57.)  $\tan \pi$

$$\boxed{0}$$

58.)  $\csc 150^\circ$

$$\boxed{2}$$

59.)  $\sin \frac{7\pi}{6}$

$$\boxed{-\frac{1}{2}}$$

60.)  $\cot 0^\circ$

$$\boxed{\text{undefined}}$$

61.)  $\sec \frac{\pi}{3}$

$$\boxed{2}$$